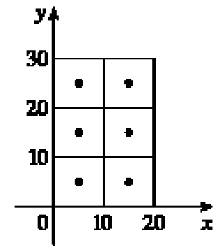
12.1: 6, 10;

6. To approximate the volume, let R be the planar region corresponding to the surface of the water in the pool, and place R on coordinate axes so that x and y correspond to the dimensions given. Then we define $f(x,y)$ to be the depth of the water at (x,y), so the volume of water in the pool is the volume of the solid that lies above the rectangle $R=[0,20]\times[0,30]$ and below the graph of $f(x,y)$. We can estimate this volume using the Midpoint Rule with $m=2$ and $n=3$, so $\Delta A=100$. Each subrectangle with its midpoint is shown in the figure. Then



$$V\approx\sum_{i=1}^{2}\sum_{j=1}^{3}f(\overline{x}_i,\overline{y}_j)\,\Delta A=\Delta A[f(5,5)+f(5,15)+f(5,25)+f(15,5)+f(15,15)+f(15,25)]$$
$$=100(3+7+10+3+5+8)=3600$$

Thus, we estimate that the pool contains 3600 cubic feet of water.

Alternatively, we can approximate the volume with a Riemann sum where $m=4$, $n=6$ and the sample points are taken to be, for example, the upper right corner of each subrectangle. Then $\Delta A=25$ and

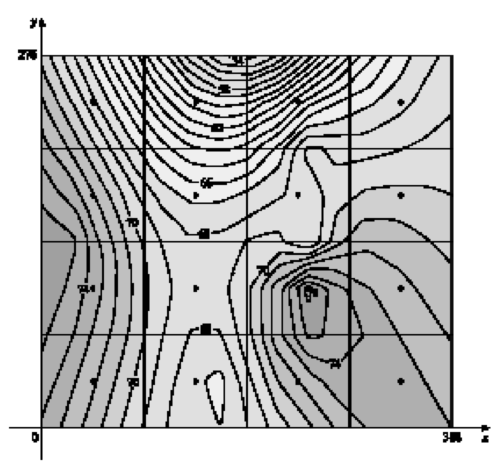
$$V\approx\sum_{i=1}^{4}\sum_{j=1}^{6}f(x_i,y_j)\,\Delta A$$
$$=25[3+4+7+8+10+8+4+6+8+10+12+10+3+4+5+6+8+7+2+2+2+3+4+4]$$
$$=25(140)=3500$$

So we estimate that the pool contains 3500 ft^3 of water.

10. As in Example 4, we place the origin at the southwest corner of the state. Then $R=[0,388]\times[0,276]$ (in miles) is the rectangle corresponding to Colorado and we define $f(x,y)$ to be the temperature at the location (x,y). The average temperature is given by

$$f_{\text{ave}}=\frac{1}{A(R)}\iint_R f(x,y)\,dA=\frac{1}{388\cdot 276}\iint_R f(x,y)\,dA$$

We can use the Midpoint Rule with $m=n=4$ to give a reasonable estimate of the value of the double integral.



12.2: 16, 22, 32

16. $\iint_R \frac{1+x^2}{1+y^2}\,dA = \int_0^1 \int_0^1 \frac{1+x^2}{1+y^2}\,dy\,dx = \int_0^1 (1+x^2)\,dx \int_0^1 \frac{1}{1+y^2}\,dy$

$= \left[x + \frac{1}{3}x^3\right]_0^1 \left[\tan^{-1} y\right]_0^1 = \left(1 + \frac{1}{3} - 0\right)\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{3}$

22. $V = \iint_R (4 + x^2 - y^2)\,dA = \int_{-1}^1 \int_0^2 (4 + x^2 - y^2)\,dy\,dx = \int_{-1}^1 \left[4y + x^2 y - \frac{1}{3}y^3\right]_{y=0}^{y=2}\,dx$

$= \int_{-1}^1 \left(2x^2 + \frac{16}{3}\right)dx = \left[\frac{2}{3}x^3 + \frac{16}{3}x\right]_{-1}^1 = \frac{2}{3} + \frac{16}{3} + \frac{2}{3} + \frac{16}{3} = 12$

32. $A(R) = 4 \cdot 1 = 4$, so

$f_{\text{ave}} = \frac{1}{A(R)} \iint_R f(x,y)\,dA = \frac{1}{4}\int_0^4 \int_0^1 e^y \sqrt{x + e^y}\,dy\,dx = \frac{1}{4}\int_0^4 \left[\frac{2}{3}(x + e^y)^{3/2}\right]_{y=0}^{y=1} dx$

$= \frac{1}{4} \cdot \frac{2}{3} \int_0^4 \left[(x+e)^{3/2} - (x+1)^{3/2}\right] dx = \frac{1}{6}\left[\frac{2}{5}(x+e)^{5/2} - \frac{2}{5}(x+1)^{5/2}\right]_0^4$

$= \frac{1}{6} \cdot \frac{2}{5}\left[(4+e)^{5/2} - 5^{5/2} - e^{5/2} + 1\right] = \frac{1}{15}\left[(4+e)^{5/2} - e^{5/2} - 5^{5/2} + 1\right] \approx 3.327$